STUDENT SUCCESS CENTER
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

MECHANICS OF MATERIALS: AXIAL LOADS & TORSION

Axial Loads:

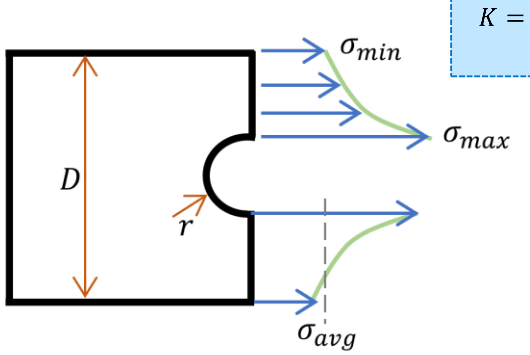
$$\varepsilon = \frac{\delta}{L} = \frac{\sigma}{\tau} = \frac{1}{E}\cdot\frac{P}{A} \quad \text{such that} \quad \delta = \frac{PL}{AE} \text{ for homogenous and uniform cross} - \text{sections}$$

$$\delta = \int_0^{L_o} \frac{P(x)}{A(x)\cdot E(x)}dx \quad = \quad \sum_{i=0}^{n}\frac{P_i L_i}{A_i E_i}$$

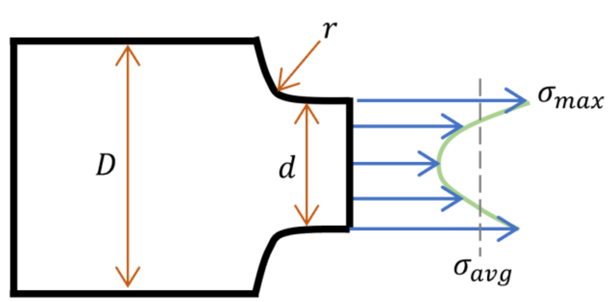
- Superposition allows the problem to be worked by evaluating how much a body *would* move if it could. Then, the reaction force must be equal and opposite.
  1. Remove one wall support
  2. Break the body into sections where P, L, A, or E changes
  3. Find δ for each section and the resulting $\delta_{total} = \sum \delta_i$ (careful to mind signs)
  4. Find $\delta_{reaction}$ in terms of the reaction force R_{wall}
  5. Equate the deformations due to the true deflection being 0 ($\delta = 0 = \delta_{no\,wall} + \delta_{reaction}$)
  6. Solve for reaction force at the wall, R_{wall}, and consequently other reaction forces.
  7. Solve for the stress in each section (from step 2), where $P = R_{wall}$ for all sections and A varies.
- Thermal Stress is a result of heating or cooling and is dependent on the material's coefficient of thermal expansion, α (found in tables, measured in $°\mathrm{F}^{-1}$ or $°\mathrm{C}^{-1}$):

$$\delta_T = \alpha L \Delta T \qquad \varepsilon_T = \alpha \Delta T$$

- Stress concentrations arise when stress flow is abruptly "pinched" around a corner. Two common instances are holes and fillets in a flat plate. This is the reason sidewalk corners crack first.



$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

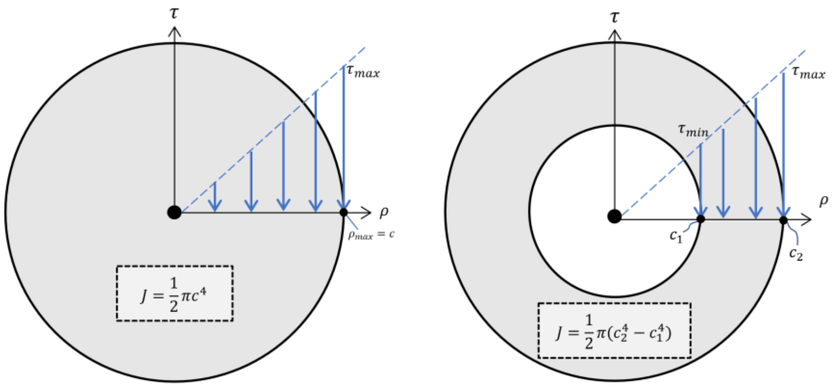


Given D, d, and r/d, K can be found using provided charts. K tells the ratio of the maximum stress observed to the average.

Torsion:

- Twisting engenders a shear stress:

$$T = \int rdF \ \ where\ \tau = \frac{dF}{dA}, thus\ T = \int r \cdot \tau dA$$

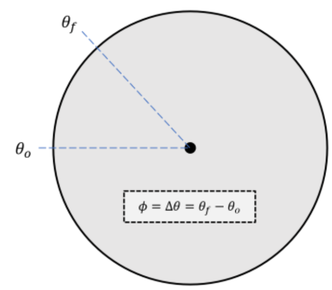


$\tau = \frac{T\rho}{J}$ for any given point on circle

$\tau_{max} = \frac{Tc}{J}$ for max stress occurring at outermost point

- Angle of twist measures deformation and is dependent on the length and diameter. Shear strain is independent of both diameter and length:

$$\gamma_{max} = \frac{c\phi}{L} = \frac{\tau_{max}}{G} = \frac{TC}{JG} \quad and \quad \phi = \frac{TL}{JG}$$



- Power transmissions can be analyzed to understand the minimum shaft diameter for a given power requirement (or max power for given shaft diameter):

$$P = t \cdot \omega = 2\pi \ \ such\ that\ \ T = \frac{P}{2\pi f} = \frac{\tau_{max} \cdot J}{c}$$

$$\frac{J}{C} = \frac{P}{2\pi f} \cdot \frac{1}{\tau_{max}}$$

$$J = \frac{TL}{G\phi} = \frac{P}{2\pi f} \cdot \frac{L}{G\phi}$$